# Formule Goniometriche:

## Formule di ADDIZIONE E SOTTRAZIONE:

$$\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \operatorname{sen}\alpha \cdot \operatorname{sen}\beta$$
$$\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha \cdot \cos\beta \pm \cos\alpha \cdot \operatorname{sen}\beta$$
$$tg(\alpha \pm \beta) = \frac{tg\,\alpha \pm tg\,\beta}{1 \mp tg\,\alpha \cdot tg\,\beta}$$

dim. appunti sul sito del prof.cantone

Dim:
$$tg(\alpha+\beta) = \frac{\operatorname{sen}(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\operatorname{sen}\alpha\cdot\cos\beta + \cos\alpha\cdot\operatorname{sen}\beta}{\cos\alpha\cdot\cos\beta - \operatorname{sen}\alpha\cdot\operatorname{sen}\beta} = \frac{\dfrac{\operatorname{sen}\alpha\cdot\cos\beta + \cos\alpha\cdot\operatorname{sen}\beta}{\cos\alpha\cdot\cos\beta}}{\dfrac{\cos\alpha\cdot\cos\beta - \operatorname{sen}\alpha\cdot\operatorname{sen}\beta}{\cos\alpha\cdot\cos\beta}} = \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha\cdot tg\,\beta}$$

## Formule di DUPLICAZIONE:

$$\operatorname{sen}2\alpha = 2\operatorname{sen}\alpha\cdot\cos\alpha$$
$$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$$
$$tg\,2\alpha = \frac{2tg\,\alpha}{1 - tg^2\alpha}$$

Dim.: $\operatorname{sen}2\alpha = \operatorname{sen}(\alpha+\alpha) = \operatorname{sen}\alpha\cdot\cos\alpha + \cos\alpha\cdot\operatorname{sen}\alpha = 2\operatorname{sen}\alpha\cdot\cos\alpha$

Dim.: $\cos 2\alpha = \cos(\alpha+\alpha) = \cos\alpha\cdot\cos\alpha - \operatorname{sen}\alpha\cdot\operatorname{sen}\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$

Dim.: $tg\,2\alpha = tg(\alpha+\alpha) = \dfrac{tg\,\alpha + tg\,\alpha}{1 - tg\alpha\cdot tg\,\alpha} = \dfrac{2tg\,\alpha}{1 - tg^2\alpha}$

## Formule PARAMETRICHE:

$$\operatorname{sen}\alpha = \frac{2tg\frac{\alpha}{2}}{1 + tg^2\frac{\alpha}{2}}$$

$$\cos\alpha = \frac{1 - tg^2\frac{\alpha}{2}}{1 + tg^2\frac{\alpha}{2}}$$

Dim.: $\operatorname{sen}2\beta = \dfrac{2\operatorname{sen}\beta\cdot\cos\beta}{1} = \dfrac{2\operatorname{sen}\beta\cdot\cos\beta}{\cos^2\beta + \operatorname{sen}^2\beta} = \dfrac{\dfrac{2\operatorname{sen}\beta\cdot\cancel{\cos\beta}}{\cos^{\cancel{2}}\beta}}{\dfrac{\cos^2\beta + \operatorname{sen}^2\beta}{\cos^2\beta}} = \dfrac{2tg\,\beta}{1 + tg^2\beta}$ ➔* $\operatorname{sen}\alpha = \dfrac{2tg\frac{\alpha}{2}}{1 + tg^2\frac{\alpha}{2}}$

Dim.: $\cos 2\beta = \dfrac{\cos^2\beta - \operatorname{sen}^2\beta}{1} = \dfrac{\cos^2\beta - \operatorname{sen}^2\beta}{\cos^2\beta + \operatorname{sen}^2\beta} = \dfrac{\dfrac{\cos^2\beta - \operatorname{sen}^2\beta}{\cos^2\beta}}{\dfrac{\cos^2\beta + \operatorname{sen}^2\beta}{\cos^2\beta}} = \dfrac{1 - tg^2\beta}{1 + tg^2\beta}$ ➔* $\cos\alpha = \dfrac{1 - tg^2\frac{\alpha}{2}}{1 + tg^2\frac{\alpha}{2}}$

## Formule di BISEZIONE:

$$\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1 + \cos\alpha}{2}}$$

$$tg\frac{\alpha}{2} = \pm\sqrt{\frac{1 - \cos\alpha}{1 + \cos\alpha}}$$

$$tg\frac{\alpha}{2} = \frac{\operatorname{sen}\alpha}{1 + \cos\alpha}$$

$$tg\frac{\alpha}{2} = \frac{1 - \cos\alpha}{\operatorname{sen}\alpha}$$

Dim.: $\cos 2\beta = \cos^2\beta - \operatorname{sen}^2\beta = 1 - \operatorname{sen}^2\beta - \operatorname{sen}^2\beta = 1 - 2\operatorname{sen}^2\beta$ ➔* $\cos\alpha = 1 - 2\operatorname{sen}^2\dfrac{\alpha}{2}$

da cui $\operatorname{sen}^2\dfrac{\alpha}{2} = \dfrac{1 - \cos\alpha}{2}$ e quindi $\operatorname{sen}\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1 - \cos\alpha}{2}}$

Dim.: $\cos 2\beta = \cos^2\beta - \operatorname{sen}^2\beta = \cos^2\beta - (1 - \cos^2\beta) = 2\cos^2\beta - 1$ ➔* $\cos\alpha = 2\cos^2\dfrac{\alpha}{2} - 1$

da cui $\cos^2\dfrac{\alpha}{2} = \dfrac{1 + \cos\alpha}{2}$ e quindi $\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1 + \cos\alpha}{2}}$

Dim.: $tg\dfrac{\alpha}{2} = \dfrac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}$ ....

Dim.: $tg\dfrac{\alpha}{2} = \dfrac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \dfrac{\operatorname{sen}\frac{\alpha}{2}\cdot 2\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}\cdot 2\cos\frac{\alpha}{2}} = \dfrac{2\operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}} = \dfrac{\operatorname{sen}\alpha}{\cancel{2}\dfrac{1 + \cos\alpha}{\cancel{2}}} = \dfrac{\operatorname{sen}\alpha}{1 + \cos\alpha}$

* Sostituisco a $2\beta$ con $\alpha$ e quindi $\beta$ con $\dfrac{\alpha}{2}$